CLASA A V-A

BAREM DE EVALUARE ȘI DE NOTARE

1 $7^d < 2023 \Rightarrow d \in \{0,1,2,3\}$ (1p)

$d=1 \Rightarrow \overline{abc} = 281 \Rightarrow 2811$ (2p)

$d=2 \Rightarrow \overline{abc} = 275 \Rightarrow 2752$ (2p)

$d=3 \Rightarrow \overline{abc} = 233 \Rightarrow 2333$ (2p)

2 $m = n = 0$ (1p)

$m>0, n=0 \Rightarrow 6^m + 1 = \ldots 7$ (3p)

$m>0, n>0 \Rightarrow 6^m + 6^n = \ldots 2$ (3p)

3 $2^3 < 3^2$ (1p) $\Rightarrow 2^{33} < 3^{22}$ (2p)

$11^2 < 5^3$ (1p) $\Rightarrow 11^{2000} < 5^{3000}$ (2p)

$a < b$ (1p)

4 $x = y + 800$ (1p)

$x = 20y + r,\ 1 \le r < y$ (1p)

$y + 800 = 20y + r \Rightarrow 19y = 800 - r$ (1p)

$19y < 800 \Rightarrow y \le 42$ (1p)

$19y > 800 - y \Rightarrow y \ge 41$ (1p)

$y = 42 \Rightarrow x = 842$ (1p)

$y = 41 \Rightarrow x = 841$ (1p)

CLASA A VI A

BAREM DE EVALUARE ȘI DE NOTARE

1 $bc = 13 \cdot 13$ (2p)

$b = 1, c = 169 \Rightarrow a = 156 \Rightarrow p = 26364$ (2p)

$b = 13, c = 13 \Rightarrow a = 12 \Rightarrow p = 2028$ (2p)

$b = 169, c = 1$ imposibil (1p)

2 $32^{11} = 2^{55}$ (1p) $\quad n = 2^k$ (2p)

$1 \cdot 2 \cdot 2^2 \cdots 2^k = 2^{55}$ (1p) $\quad k(k+1)/2 = 55$ (1p)

$k = 10$ (1p) $\quad n = 2^{10} = 1024$ (1p)

3 $1331 = nq_1 + 11,\ n \geq 12$ (2p)

$349 = nq_2 + 13,\ n \geq 14$ (2p)

$nq_1 = 1320$; $nq_2 = 336$ (1p)

$n \geq 14$ este divizor comun al lui

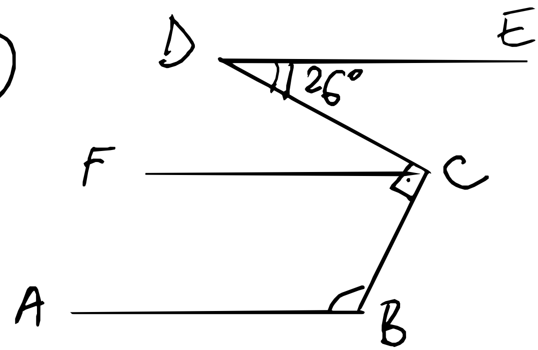
1320 și 336 (1p) $\Rightarrow n = 24$ (1p)

4 $FC \parallel DE$ (1p)

$m(\sphericalangle DCF) = 26°$ (2p)

$m(\sphericalangle FCB) = 64°$ (2p)

$m(\sphericalangle ABC) = 116°$ (2p)



CLASA A VII-A

BAREM DE EVALUARE ȘI DE NOTARE

(1) $xy+1=2y$ (1p) $\frac{1}{z}=xy$ (1p)

$\Rightarrow y+xy=3$ (1p) $2y-1=3-y\ (=xy)$ (1p)

$\Rightarrow y=4/3$ (1p) $x=5/4$ (1p) $z=3/5$ (1p)

(2) $S=\left[1+\frac{\sqrt{1}}{1}\right]+\left[1+\frac{\sqrt{2}}{2}\right]+\dots$ (3p)

$=2+1+\dots+1$ (3p) $=11$ (1p)

(3) $a=\sqrt{m-2}\in\mathbb{N}$, $b=\sqrt{2-n}\in\mathbb{N}$ (1p)

$a=0,\ b=2\Rightarrow m=2,\ n=-2$ (2p)

$a=1,\ b=1\Rightarrow m=3,\ n=1$ (2p)

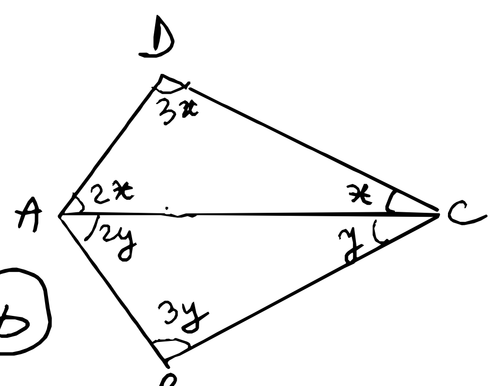
$a=2,\ b=0\Rightarrow m=6,\ n=2$ (2p)

(4) În ΔADC: $x, 2x, 3x$ (1p)

În ΔABC: $y, 2y, 3y$ (1p)

$x=y=30^\circ$ (1p)

$\hat{D}=\hat{B}=90^\circ\Rightarrow ABCD$ înscr (1p)



$\Delta ABC\equiv\Delta ADC$ (1p)

$\Rightarrow AB\equiv AD,\ CD\equiv CB$ (1p) $\Rightarrow AD+BC=AB+CD$ (1p)

CLASA A VIII-A

BAREM DE EVALUARE ȘI DE NOTARE

1) $\frac{ax+b}{c} = \dots = \frac{\sum(ax+b)}{\sum c} = x+1$ (2p)

$ax+b = cx+c \Rightarrow (a-c)x = c-b$ (1p)

$bx+c = ax+a \Rightarrow (b-a)x = a-c$ (1p)

Dacă $a<c \Rightarrow c<b$ și $b<a$ (F) (1p)

Dacă $a>c \Rightarrow c>b$ și $b>a$ (F) (1p)

Rezultă $a=c$ $(=b)$ (1p)

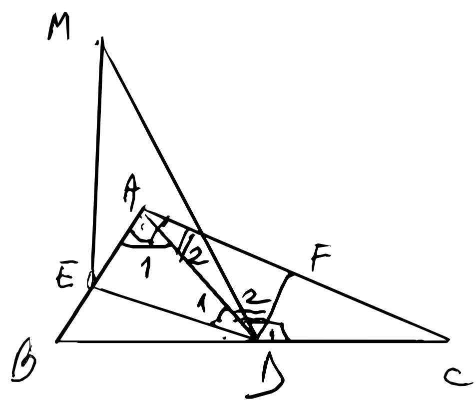
2) $(x-3)^2+(y-4)^2=25$ (3p)

$(x-3)^2 \le 25$ (1p) $-5 \le x-3 \le 5$ (1p)

$(y-4)^2 \le 25$ (1p) $-5 \le y-4 \le 5$ (1p)

3) $2r \in \mathbb{Z}$ (3p) $r = 11r - 10r \in \mathbb{Z}$ (3p)

$r = 0, -1$ sau 1 (1p)



4) DE mediană în ΔADB

$(\widehat{ADB} = 90^\circ) \Rightarrow \hat{D}_1 = \hat{A}_1$ (2p)

Analog $\hat{D}_2 = \hat{A}_2$ (2p)

$EDF = 90^\circ$ (1p)

$ME \perp (ABC)$, $ED \perp DF \xrightarrow{3\perp} MD \perp DF$ (2p)

CLASA A IX A

BAREM DE EVALUARE ȘI DE NOTARE

1 $\left([x]-\{x\}\right)\left(1-\dfrac{2023}{[x]\{x\}}\right)=0$ (2p)

I) $[x]=\{x\} \Rightarrow x=0$ (2p)

II) $[x]\cdot\{x\}=2023 \Rightarrow \{x\}=\dfrac{2023}{k}$, $k=[x]$ (1p)

$k \geq 2024$ (1p) $x=k+\dfrac{2023}{k}$, $k\geq 2024$, $k\in\mathbb{N}$ (1p)

2 $14=3+3+3+8$; $15=3+3+3+3+3$;

$16=8+8$ (1+1+1p)

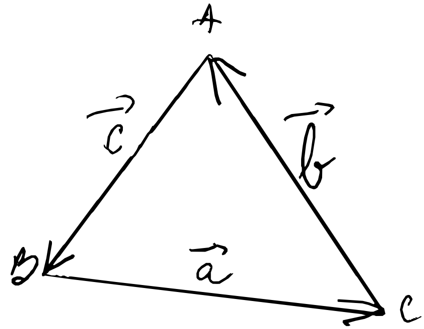
$n=k_1+\dots+k_p \Rightarrow n+3=k_1+\dots+k_p+3$ (3p)

Finalizare (1p)

3 $E \geq \dfrac{(a+b+c)^2}{\sum(a^2+bc+ca)}$ (3p)

$=\dfrac{(a+b+c)^2}{a^2+b^2+c^2+2ab+2bc+2ca}$ (3p) $=1$ (1p)

4 $\vec{m}_A=\frac{1}{2}(\vec{c}-\vec{b})$ etc (2p)



$BC \parallel m_{A_1} \Rightarrow x\vec{a}=\vec{c}_1-\vec{b}_1$

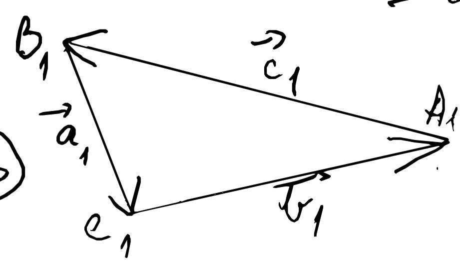
$CA \parallel m_{B_1} \Rightarrow y\vec{b}=\vec{a}_1-\vec{c}_1$

$AB \parallel m_{C_1} \Rightarrow z\vec{c}=\vec{b}_1-\vec{a}_1$

cu $x,y,z\in\mathbb{R}^*$ (2p)

$x\vec{a}+y\vec{b}+z\vec{c}=0$

$\Rightarrow x=y=z$ (1p)



$x\vec{c}-x\vec{b}=\vec{b}_1-\vec{a}_1-(\vec{a}_1-\vec{c}_1)=-3\vec{a}_1$ (1p)

$\Rightarrow \vec{c}-\vec{b}=\dfrac{-3}{x}\vec{a}_1$ (1p) etc

CLASA A X-A

BAREM DE EVALUARE ȘI DE NOTARE

1 $(y-1)(y-2)(y-3)(y-4)=-1$, $y=\lg x$ (2p)

$(y^2-5y+4)(y^2-5y+6)+1=0$ (2p)

$(t-1)(t+1)+1=0$, $t=y^2-5y+5$ (2p)

$t=0$, Finalizare (1p)

2 $z=a+bi$, $a^2+b^2=1$ (1p)

$z^2+\bar{z}^2=2(a^2-b^2)$ (1p) $\Rightarrow a^2-b^2=\pm 1$ (1p)

$z=1, -1, i$ sau $-i$ (1+1+1+1 p)

3 a) $2^x+2^{-x}=6$ (1p) $2^x=t$, $t+\frac{1}{t}=6$ (1p)

Finalizare (1p)

b) $E \geq (1+2^x)^3+(1+2^{-x})^3$ (2p)

$=(8^x+8^{-x})+3(4^x+4^{-x})+3(2^x+2^{-x})+2 \geq 16$ (2p)

4 a) $h'(0)=0$ (1p) $h(2)=0$ (1p)

$h(0)=h(2)$ (1p)

b) $x^3+ax=y^3+ay$ (1p)

$(x^3-y^3)+a(x-y)=0$ (1p)

$(x-y)(x^2+xy+y^2+a)=0$ (1p) $\Rightarrow x=y$ (1p)

CLASA A $\overline{XI}$ A

BAREM DE EVALUARE ȘI DE NOTARE

(1) $\{\sqrt{n^2+n}\} = \sqrt{n^2+n} - n$ (2p)

$= \dfrac{n}{n+\sqrt{n^2+n}}$ (2p) $\longrightarrow \dfrac{1}{2}$ (3p)

(2) $0 < f(x) < x$ (2p) $\Rightarrow \alpha = 0$ (1p)

$f(x) = \dfrac{-1+\sqrt{4x+1}}{2}$ (2p) $\Rightarrow \beta = 1$ (2p)

(3) $\det(I_2 + iA) \cdot \det(I_2 - iA)$ (2p)

$= z \cdot \overline{z}$ (1p) $= |z|^2 \geq 0$ (1p)

$\det(I_2 + A) \cdot \det(I_2 - A)$ (2p)

$= \det(I_2 + A) \cdot \det(I_2 + A)^t = \det^2(I_2 + A) \geq 0$ (1p)

(4) a) $\det A = 0$ (1p) $A^2 = \lambda A$ (1p)

$A^4 = \lambda^3 A$ (1p) $\lambda = 0$ sau $A = O_2 \Rightarrow A^2 = O_2$ (1p)

b) $X^4 = X^2 \cdot X^2 = O_2$ (2p) $\overset{a)}{\Rightarrow} X^2 = O_2$ (F) (1p)

CLASA A XII A

BAREM DE EVALUARE și DE NOTARE

(1) $(1/f)' = f$ (1p) $-f'/f^2 = f$ (1p)

$-f'/f^3 = 1 \Rightarrow \left(\frac{1}{2f^2}\right)' = 1$ (2p)

$\frac{1}{2f^2(x)} = x + c$ (1p) $f(x) = \frac{1}{\sqrt{2(x+c)}}$ (1p)

unde $c \geq 0$ (1p)

(2) $M = \sup\limits_{[0,1]} \frac{2x+3}{x(x+1)(x+2)(x+3)+1} < \infty$ (2p)

$0 \leq I_n \leq \int_0^{1/n} M\,dx$ (3p) $= \frac{M}{n} \to 0$ (1p)

$\Rightarrow L = 0$ (1p)

(3) a) $x^2y^2 = (xy)^2$ (2p) $\Rightarrow xy = yx$ (2p)

b) Dacă $p > 2$ prim, $p \mid \operatorname{ord} G$ (1p) $\Rightarrow$

există un element x cu $\operatorname{ord} x = p$ (F) (2p)

(4) i) $\Rightarrow$ ii) PRA $xy \in H$. (1p) $\Rightarrow y = x^{-1} \cdot (xy) \in H$

fals (2p)

ii) $\Rightarrow$ i) PRA $e \in G \setminus H$. Pentru $x \in H \Rightarrow e \cdot x \in G \setminus H$

$\Rightarrow x \in G \setminus H$ ↯ (2p)

PRA $\exists x \in H$, $x^{-1} \in G \setminus H \Rightarrow x \cdot x^{-1} \in G \setminus H$

$\Rightarrow e \in G \setminus H$ (F) (2p)